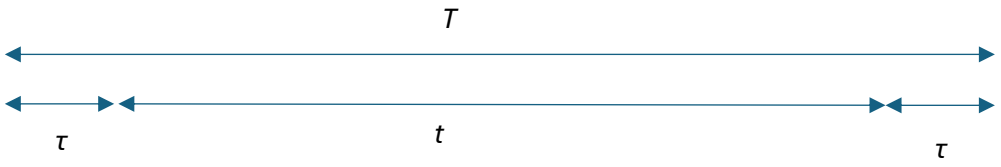
Teacher notes
Topic A

An impulse problem.

A man of mass M stands on weighing scales. At $t = 0$ he throws a ball of mass m vertically upwards. The ball leaves his hand after a short time τ . The ball reaches a height h spending time t in the air before returning to his hand. It again takes time τ to catch the ball and bring it to rest. The whole process takes time T.



(a) What does the scale read when the ball is in the air?

(b) What is the impulse (magnitude and direction) provided to the scales while
  (i) the ball is being thrown upwards?
  (ii) the ball is being caught on the way down?

(c) What is the average reading of the scale during the time from $t = 0$ to $t = T$?

Answers

(a) With the ball in the air the only mass on the weighing scales is *M* and so the reading is *Mg*.

(b)

(i) The man imparts an impulse *mv* to the ball when throwing it upwards. The speed of the ball is $\sqrt{2gh}$ and so the impulse delivered to the scales downwards is $m\sqrt{2gh}$.

(ii) The ball received also exerts a downward impulse on the scales and so the answer is the same as in (ii), $m\sqrt{2gh}$.

(c) The ball is in the air for a time *t* given $t = 2\sqrt{\frac{2h}{g}}$. During this time the only mass on the scales is *M*. For a time $T - t$ the mass on the scales is *M* + *m*. During the time *T* the total impulse delivered to the scales by throwing and catching the ball is $2m\sqrt{2gh}$. Therefore, the time average of the scales reading *F* is

$$F = \frac{Mgt + (M+m)\,g(T-t)}{T} + \frac{2m\sqrt{2gh}}{T}$$

$$F = \frac{MgT + mgT - mgt + 2m\sqrt{2gh}}{T}$$

$$F = (M+m)\,g + \frac{-mgt + 2m\sqrt{2gh}}{T}$$

The time of flight of the ball is $t = 2\sqrt{\frac{2h}{g}}$ and so the last term in the force is

$$\frac{-mgt + 2m\sqrt{2gh}}{T} = \frac{-mg2\sqrt{\frac{2h}{g}} + 2m\sqrt{2gh}}{T} = 0.$$

Hence the time averaged reading is just $F = (M+m)\,g$.